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FIELD THEORY OF LOCALIZATION AND SUPERCONDUCTIVITY *

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ABSTRACT

A field theory of both localization and superconductivity is constructed by introducing a new matrix field describing the superconductivity.

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1. INTRODUCTION

The field theory of localization [1]-[3] was constructed in order to reach a non-perturbative description of the Anderson transition [4] driven by the disorder. It is a metal-insulator type transition described by the Wegner's[1] order parameter Q introduced initially in replica based theories. Recently, two replica free theories were constructed [5],[6] by Efetov [5] employing the supersymmetry and by Aronov and Ioselevich [6] with a Feynman type functional integral. The intensive research taking into account the electron-electron interaction revealed a strong coupling between the disorder and the interaction [7]-[10] making the Anderson transition inseparable from the Mott transition.

Here we develop a field theory of localization for simplicity in the replica formulation of Finkelstein [3]. The theory accounts for the superconducvitity by another order parameter - a matrix field corresponding to the pairs wave function. In this form the theory reproduces the results of the perturbation theory of Abrikosov and Gor'kov [11] and takes into account the diffusion. In general it is a non-perturbative theory of both phase transitions.

2. FREE ENERGY FUNCTIONAL

We stress again here that the theory can be constructed in a replica free manner, but for simplicity we use the replica trick. Then the calculation of the random impurities average of the free energy is simple

$$\langle\langle F \rangle\rangle = -T \langle\langle \ln Z \rangle\rangle = -T \lim_{N\to 0} \langle\langle \frac{Z^N - 1}{N} \rangle\rangle \qquad (1)$$

The double angular brackets here denote average over the impurities potential $V(r)$. For the calculation of $\langle\langle Z_N \rangle\rangle = \langle\langle Z^N \rangle\rangle$ we use the standard thermodynamic field theory introducing thermal frequencies $\varepsilon_n = (2n + 1)\pi T$ and imaginary time τ $(0 < \tau < \beta)$. The classical Grassmann variables are used in order to obtain correct statistics. The Grassmann field $\psi^{i\alpha}(r,\varepsilon_n)$ has a replica index $i = 1,2,\ldots,N$, a spin index $\alpha = 1, 2$ and a discrete energy argument ε_n. The partition function of N replicas of the system of N_0 electrons in the presence of impurity potential $V(r)$ has the form

$$Z_N = \int e^{S_N} \, d\bar{\Psi} \, d\Psi \tag{2}$$

Here the differential $d\phi$ denotes the product of all components of $d\phi$ and functional integration over each one. The action S_N is

$$S_N = \sum_{i,n,\alpha} \int \left[\bar{\Psi}^{i\alpha}_{(\tau,\varepsilon_n)} \left(i\varepsilon_n - \hat{\mathcal{H}}_i(\tau) \right) \Psi^{i\alpha}_{(\tau,\varepsilon_n)} \right] d\tau + S^{el} \tag{3}$$

The Hamiltonian of the non-interacting electrons is

$$\mathcal{H}_i = \frac{p^2}{2m} - \mu + V(\tau) = H_0(\tau) + V(\tau) \tag{4}$$

The term S^{el} represents the electron-electron interaction discussed in detail by Finkelstein [3],[8] and the Rome group [9],[10].

The impurity potential $V(r)$ is assumed to be a short range one with Gaussian distribution. The correlation function of the potential is of white noise type

$$\langle\langle V(\tau) V(\tau') \rangle\rangle = \frac{1}{2\pi\nu\tau} \, \delta(\tau - \tau') \tag{5}$$

Here ν is the Fermi level density of states and τ is the relaxation time. The average over the impurities potential lead to the result

$$\langle\langle Z_N \rangle\rangle = \int e^S \, d\bar{\Psi} \, d\Psi \tag{6}$$

Introducing the symbol $\hat{\varepsilon}$ for the diagonal frequencies matrix one can write S in the form

$$S = \int \left[\langle \bar{\Psi} (i\varepsilon - H_0) \Psi \rangle + \frac{\langle \bar{\Psi}\Psi \rangle^2}{4\pi\nu\tau} \right] d\tau + S^{el} \tag{7}$$

The angular brackets denote scalar product

$$\langle \bar{\Psi}\Psi \rangle = \sum_{i,n,\alpha} \bar{\Psi}^{i\alpha}(\tau,\varepsilon_n) \, \Psi^{i\alpha}(\tau,\varepsilon_n) \tag{8}$$

The fourth order term in S can be converted by Hubbard-Stratanovich transformation of the type

$$e^{\frac{1}{4\pi\nu\tau}\int \langle \bar{\Psi}\Psi \rangle^2 d\tau} \int e^{-\frac{\pi\nu}{4\tau}\int \mathrm{Tr}(Q^2) d\tau} \, dQ = \int e^{-\frac{\pi\nu}{4\tau}\int \mathrm{Tr}\, Q^2 d\tau + \frac{i}{2\tau}\int \langle \bar{\Psi} Q \Psi \rangle d\tau} \, dQ \tag{9}$$

The Wegner Q matrix field defined by (9) is Hermitian. In terms of Q the action (7) is written as

$$S(\Psi, Q) = \int d\tau \left\{ \langle \bar{\Psi} \cdot (i\hat{\varepsilon} - H_0) \Psi \rangle - \frac{i}{2\tau} \langle \bar{\Psi} Q \Psi \rangle - \frac{\pi\nu}{4\tau} \mathrm{Tr}(Q^2) \right\} \tag{10}$$

It is seen from (10) that the diagonal matrix $(\hat{\varepsilon})_{n,m} = \delta_{n,m}(2n + 1)\pi T$ is now modified to a new matrix $\varepsilon \cdot \hat{\eta}$, where $\hat{\eta}$ is defined as

$$\hat{\eta} = 1 + \frac{\varepsilon^{-1} Q}{2\tau} \tag{11}$$

This remark is very important since in this way a new field $\hat{\eta}(r,\tau)$ is introduced in the same way as the parameter η was introduced by Abrikosov and Gor'kov. At this stage integrating over the Grassmann variables $\bar{\psi}(r,\tau)$, $\psi(r,\tau)$ one can reach the diffusion Lagrangian without interaction. The interaction can be taken into account in the manner used by Finkelstein [3],[8] and the Rome group [9],[10]. From such a scheme a full scale theory of the normal charged Fermi liquid in the presence of disorder is constructed. Our aim, however, is to analyze the phenomena of superconductivity and localization in relation to each other.

3. LOCALIZATION AND SUPERCONDUCTIVITY

The electron-electron interaction responsible for pairing will be taken in the BCS form

$$S^{el} = |g| \int dr \int_0^\beta d\tau \, \langle \bar{\Psi}(r,\tau)\Psi(r,\tau)\rangle^2 \tag{12}$$

In order to take into account the anomalous propagator this expression is transformed by another matrix field $\dagger(r,\tau)$ which is complex and antisymmetric according to the relation

$$\dagger_{i\alpha,j\beta}(r,\tau) = -\dagger_{j\beta,i\alpha}(r,\tau)$$

The new Hubbard-Stratonovich transformation is

$$e^{|g|\iint_0^\beta \langle\bar{\Psi}(r,\tau)\Psi(r,\tau)\rangle^2 dr\,d\tau} \cdot \int e^{-\frac{1}{|g|}\int Tr(\bar{\dagger}.\dagger)\,dr\,d\tau}\, d\bar{\dagger}\, d\dagger =$$
$$= \int e^{-\int\left\{\frac{1}{|g|}Tr(\bar{\dagger}.\dagger) - \langle\Psi.\bar{\dagger}.\Psi\rangle - \langle\bar{\Psi}.\dagger.\Psi\rangle\right.}\, d\dagger\, d\dagger \tag{13}$$

The functional integration is carried over all components of the antisymmetric matrices $\dagger$, $\bar{\dagger}$ ($\bar{\dagger} = \dagger^{*T}$). The transformation (13) of the partitition function leads to the following action:

$$S = \int dr \int_0^\beta d\tau \left\{ \langle\bar{\Psi}.(i\hat{\varepsilon}\hat{\eta} - \hat{H}_0(r)).\Psi(r,t)\rangle - \frac{\pi}{4\tau}Tr(Q^2) - \frac{1}{|g|}Tr(\bar{\dagger}\dagger) + \langle\Psi\bar{\dagger}\Psi\rangle + \langle\bar{\Psi}\dagger\bar{\Psi}\rangle \right\} \tag{14}$$

The integral over the Grassmann variables $\psi_{(r,\tau)}$, $\bar{\psi}_{(r,\tau)}$ is no longer exact and the free energy loops have to be calculated. In the case of superconductivity only the first loop is necessary leading to the Gor'kov equations. In this way the field theory of both localization and superconductivity is determined in the form

$$Z_N = \int e^{-\mathfrak{F}(Q,\bar{\dagger},\dagger)}\, dQ\, d\bar{\dagger}\, d\dagger \tag{15}$$

The free energy functional is

$$-\mathfrak{F} = \int Tr\left\{ \ln\left[(\mathcal{H}\bar{\mathcal{H}} - \bar{\dagger}^{T}\dagger)\cdot\bar{\mathcal{H}}^{-1}\right] - \frac{\pi\nu}{4\tau}Q^2 - \frac{1}{|g|}\bar{\dagger}.\dagger \right\} dr \tag{16}$$

Here $\mathcal{H}$ and $\bar{\mathcal{H}}$ are

$$\mathcal{H} = i\hat{\varepsilon}\hat{\eta} - \hat{H}_0(r) \quad ; \quad \bar{\mathcal{H}} = -i\hat{\varepsilon}\hat{\eta} - \hat{H}_0(r) \tag{17}$$

Minimizing $\mathfrak{F}$ one finds the following equilibrium solution for the matrix η:

$$\hat{\eta} = 1 + \frac{\eta}{\pi\nu\tau}\int\frac{dp}{(2\pi)^d}\sum_n \frac{1}{\varepsilon_n^2\eta^2 + \xi_p^2 + \Delta^2} \tag{18}$$

where $\xi_p = \varepsilon_p - \mu$ and $-\bar{\Phi}^T \cdot \Phi = +\bar{\Phi} \cdot \Phi = \Delta^2$ determines the gap. The gap value is found from the BCS equation obtained by minimization of $\mathcal{F}$ with respect to the matrix Φ. In the presence of paramagnetic impurities these equations are modified leading to the critical temperature shift found first by Abrikosov and Gor'kov. In this sense the description of both localization and superconductivity by the two fields $Q(r,\tau)$ and $\Phi(r,\tau)$, $\bar{\Phi}(r,\tau)$ is complete as defined by (15)-(17). The Ginsburg-Landau-Wilson Hamiltonian is easily found from the fixed point solutions and the results will be published elsewhere.

4. CONCLUSIONS

In conclusion we have constructed a field theory describing both transitions metal-insulator and superconductivity. The theory is represented in a replica based form in terms of the Wegner hermitian matrix field $Q(r,\tau)$ and two complex antisymmetric matrix fields $\bar{\Phi}(r,\tau)$, $\Phi(r,\tau)$.

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